The Effect of Notches on the Capacity of Wood Columns

Daniel Merrick, PE

Dan@danielmerrick.com

001.408.778.9272

001.408.778.9168 fax

Department of Civil and Environmental Engineering

San Jose State University

One Washington Square

San Jose, CA 95192-0083

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Abstract

There is no established method known to the author that can be used to predict the buckling or crushing capacity of columns that have a notch or reduced cross section at some point along its length. A method for calculating the capacity of long, intermediate and short wood columns that have a notched or reduced section is presented. A series of column tests was performed to validate the method.

1 Introduction

During a recent failure investigation I was called upon to evaluate the premature collapse of a structure in which columns had been intentionally weakened in order to facilitate engineered demolition. During this investigation it became apparent that within the engineering literature there is no established method for determining the capacity of compression members that have a reduced section over a small portion of their length at or near the point of maximum curvature. Also, in timber construction it is not uncommon for notches to be cut in members to facilitate joints or for the passage of mechanical and electrical components of the structure. Weakening steel columns by notching or partial cutting is not uncommon in demolition. This study was performed in order to explore methods of predicting the capacity of columns with notches.

It is anticipated that established standard equations for the determining the capacity of columns can be adapted and used to predict the capacity of columns with a reduced section over a small portion of their length. The design equation chosen would be based on the un-notched section properties for the elastic terms in the equations and on the notched section properties for the inelastic or strength terms in the equations.

The current design equation for wood columns is a form of the Ylinen equation which determines the capacity of a column on the basis of the crushing strength of the section (compressive strength multiplied by the area of the cross section) and the elastic strength of the column as predicted by Euler's equation. In a notched column, the crushing strength is reduced in proportion to the reduced area of the cross section while the elastic strength can be assumed to be unchanged.

2 Background

In 1759 Leonard Euler published his classic Sur la Force des Colonnes (Concerning the strength of columns) which provided the first practical method of predicting the capacity of columns. However, column tests indicated that Euler's solution did not apply to all compression elements. In 1840, Thomas Tredgold published the results of a series of tests by Eaton Hodgkinson on columns made of various metals and woods that verified Euler's predictions for "long pillars" but found that "short flexible pillars" varied substantially from Euler's prediction.

As the industrial revolution progressed it became critical for engineers to develop a design method for predicting the capacity of non-Euler columns. In about 1866 the Rankine-Gordon equation, which seems to be a modification of an equation proposed by Tredgold, had appeared. The Rankine-Gordon formula is a simple interaction formula limited by the Euler strength and the crushing strength. By the late 19^{th}

century a large body of test data was available and in 1886 Thomas H. Johnson published a paper summarizing the results and the equations proposed to date. T.H. Johnson's summary indicates that engineers had not yet solved the problem of designing non-Euler columns and were still struggling with the effects of end conditions on column capacity. In an article published in the December 22, 1888 edition of Engineering News, T.H. Johnson relates that one colleague "seemed to think that all columns were imbued with the spirit of total depravity." The colleague further explained his opinion with "The platted results being scattered over the chart like the stars above, and with about as much, or as little regularity." T.H. Johnston proposed a straight line equation which was tangent to Euler's equation with an intercept at the crushing strength. T.H. Johnson's straight line solution was overly conservative. However, in 1893 John Butler Johnson noticed that the portion of the data that deviated from Euler's solution had roughly the form of a parabola tangent to Euler's equation with an apex at the crushing strength. J.B. Johnson, who wrote the standard structural engineering text of the day, published the parabolic equation he developed in his textbook. In part due to the popularity and longevity of his textbook, J.B. Johnson's parabolic equation became and still is the standard equation for the design of short and intermediate length steel columns. In solving for the two unknown terms in the J.B. Johnson parabolic equation, one sets the parabolic equation equal to the crushing strength for a length of zero to find the first constant and equates the derivatives of the parabola and Euler's equation to find the second constant. See Figure 1.

While the equations for the design of steel columns had become standardized by the end of the 19th century, wood column design methods remained undeveloped. By the time the first national building codes were published in the 1920's, wood column capacities were determined from tabulated values rather than equations. In 1930, the United States Department of Agriculture published Bulletin No. 167 which presented the results of a timber column testing program and also proposed design equations. The USDA researchers had found that a plot of their data for intermediate columns had a sharper curvature than was represented by J.B. Johnson's parabolic second order equation and they proposed a similar fourth order equation. Note that the form of Johnson's equation can be solved for any exponent resulting in a range of curvatures. The 1st order solution is T.H. Johnson's straight line.

The USDA 4th order equation remained the basis for wood design until the 1991 National Design Specification which provided a single equation for columns of any slenderness rather than the two equations used in the USDA based design method. The 1991 NDS equation was based on work by the Finnish engineer Arvo Albin Johannes Ylinen that he had published in 1956. Ylinen had looked at a simple interaction equation, similar to the Rankine-Gordon equation, and noticed that simple interaction did not match the data very closely. He made two changes to the simple interaction equation; he added a term which modified the interaction and he modified the Euler portion of the equation to simulate a non-proportional elastic material similar to the tangent modulus approach. Both of these adjustments are made with the single constant "c" in his equation. The appropriate value of c is a variable of the stress strain behavior of the material, the shape of the cross section and any defects such as straightness, knots, etc. The value of c is found empirically from either test data or by matching the results of the equation to earlier established methods. The Ylinen equation is very flexible and by adjusting the coefficient c it can be made to approximate a wide variety of data sets. Since it is an interaction equation involving Euler's equation, the Ylinen equation returns a value less that Euler's equation for any length which reflects the fact that Euler's equation tends to slightly over-estimate the capacity of long columns. If the Ylinen coefficient is 1.00, no interaction or reduction of Euler's solution or the crushing strength occurs. See Figure 2.

Euler's elastic buckling solution is based on the lateral deflection of the column due to the bending moment which in turn is caused by the deflection eccentricity. This elastic defection curve is not significantly affected by a reduction in the moment of inertia if that reduction in the moment of inertia only occurs over a short length. The same is true for beam deflection; if the moment of inertia of a beam is reduced over a short length of the beam, the deflection of the beam is not significantly influenced. The strength of the notched beam may be significantly reduced but the stiffness is not.

H. Liebowitz published a series of papers presenting the results of notched aluminum column tests. His interest was fracture and he therefore tested the columns with eccentric loading in order to cause tensile stresses and promote fracture. He investigated the relationship between notch size and root radius in regard to failure by fracture in high strength aluminum alloy. His area of inquiry is not directly related to this study. However, some of his results do indicate that a notch does not significantly affect the capacity of a long slender column.

For the columns without a notch, Ylinen's equation for critical load is:

$$P_{cr} = \frac{P_c + P_e}{2c} - \sqrt{\left(\frac{P_c + P_e}{2c}\right)^2 - \frac{P_c P_e}{c}}$$

$$P_c = F_c A$$

$$P_e = \frac{\pi^2 EI}{{l_e}^2}$$

Where:

A	= Area of section
c	= Empirical coefficient in Ylinen's equation
d	= Minimum dimension of rectangular section
F_c	= Crushing stress
I	= Moment of Inertia
l	= Effective length of column
P_c	= Crushing strength of a short block
P_{cr}	= Critical Load
P_e	= Buckling strength provided by Euler's equation

r = Radius of Gyration $(I/A)^{1/2}$

3 Materials and Method

Yellow Poplar was selected as the test material due to its commercial availability, uniformity of properties and the fact that clear specimens are easy to obtain. Commercially milled, 1.50 inch by 0.81 inch (3.81 cm x 2.06 cm) pieces in 16 foot (4.88 m) length were acquired at a local building supply store. The pieces had been stored indoors at the building supply and all appeared to be from the same source. Each test piece of a given length was cut from a different sample in an effort to randomize selection. Four specimens of each configuration were fabricated. Columns for the specimens without notches were 10, 14, 18, 22, 25.9 and 30 inches (25.4, 35.6, 36.8, 44.5, 57.2, 76.2, 87.6 cm) long. The notched specimens were 5, 10, 14.5, 17.5, 22.5, 26.5, 30 and 34.5 inches (12.7, 25.4, 36.8, 44.5, 57.2, 67.3, 76.2, 87.6 cm) long. These lengths were selected in an effort to bracket the expected inflection point in the critical load function. For the notched specimens, a rectangular groove 0.5 inch (1.27 cm) wide was routed in each wide face leaving a net thickness of about 0.52 inch (1.32 cm). The notch resulted in a 35% reduction in the cross sectional area and a 73% reduction in the moment of inertia.

In order to test the pieces in a pin-pin condition, 1.50 inch (3.81 cm) long sections of 0.75 inch (1.91 cm) diameter half-cylindrical pieces of steel were acquired. The contact force on the half-cylinders is assumed to be radial, therefore the contact force is directed at the centerline of the cylinder and the effective length of the test specimen is the length of the test specimen. See Figures 3 and 4.

Several crush blocks were also tested without the half-cylindrical end caps in order to test with the smallest effective length possible and estimate the crushing stress for the material. The effective length of these specimens is assumed to be half the tested length. Four 2 inch (5.08 cm) long and four 5 inch (12.7 cm) long specimens were tested without notches. See Figure 5.

Each specimen was placed in a universal testing machine between parallel fixed load plates, checked for plumb and loaded at a constant rate of displacement. All specimens were tested at the same deflection rate meaning that the shorter specimens were tested at a proportionally higher strain rate.

4 Theory

For the columns with a notch, the proposed version of Ylinen's equation for critical load is:

$$P_{crn} = \frac{P_{cn} + P_e}{2c} - \sqrt{\left(\frac{P_{cn} + P_e}{2c}\right)^2 - \frac{P_{cn}P_e}{c}}$$

$$P_{cn} = F_c A_n$$

Where:

A_n = Area of section at notch

P_{cn} = Crushing strength of a short block with a notch

5 Results

The results of the tests of columns with and without notches are presented in figure 6.

Columns Without Notches				**Columns With Notches**			
Effective Length		Ave. Critical Load		Effective Length		Ave. Critical Load	
Inches	cm	Pounds	kN	Inches	cm	Pounds	kN
1	2.54	10543	46.90	5	12.70	5843	25.99
2.5	6.35	9883	43.96	10	25.40	4778	21.25
10	25.40	6740	29.98	14.5	36.83	3553	15.80
14	35.56	5698	25.34	17.5	44.45	2678	11.91
18	45.72	3548	15.78	22.5	57.15	1815	8.07
22	55.88	2393	10.64	26.5	67.31	1364	6.07
25.9	65.79	1682	7.48	30	76.20	1055	4.69
30	76.20	1412	6.28	34.5	87.63	804	3.57

6 Discussion

The results clearly demonstrated that a small, symmetric notch did not significantly affect the capacity of long, Euler columns while significantly reducing the capacity of short and intermediate columns. The critical load verses length plot (Figure 6) shows some irregularity but clearly shows the expected reverse in curvature for columns with and without notches. The irregularity of the data, particularly the 14 inch (35.6 cm) columns without notch, might be attributed to the small sample size and inadequate randomization of samples.

In calculating the theoretical values for both the notched and un-notched columns, the crushing strength, the modulus of elasticity and the Ylinen constant "c" were empirically selected. A crushing strength of 8790 psi (60.6 MPa) was selected in order to match the result of the short crush block tests. A modulus of elasticity of 1.96×10^6 psi (13.5 GPa) was chosen so that Euler's equation matched the results from the longest non-notched column tests. An Ylinen constant of 0.50 was selected for a good fit to the non-notched column data. For comparison, the Forest Products Laboratory Wood Handbook lists the modulus of elasticity of Yellow Poplar as 1.58×10^6 psi (1.09 GPa) and the crushing strength (compression parallel to the grain) as 5540 psi (38.2 MPa). The FPL values are based on a moisture content of 12% and our specimens were approximately 6%.

7 Conclusion

These results indicate that Ylinen's equation for the capacity of wood columns can be adapted to determine the capacity of wood columns with a notch of any slenderness. The adaptation needed is to simply reduce the cross sectional area used in the equation to the cross sectional area at the notch. In practical application, the standard design equation with factor of safety provided in the NDS can be adapted and used.

This work was initially motivated by the collapse of a large steel structure. A program of testing to investigate steel should be developed and implemented. Steel would arguably be a better research material due to more consistent material properties and the virtually ideal stress-strain behavior.

It is arguable that most columns in service are not slender enough to be considered Euler columns. Future research should focus on developing methods to predict the capacity of non-elastic columns. Based on the data in this study and unpublished steel data, it is anticipated that some form of Johnson's parabolic equation will be best suited to steel columns with notches and that some form of Ylinen's equation will be best suited to wood columns with notches.

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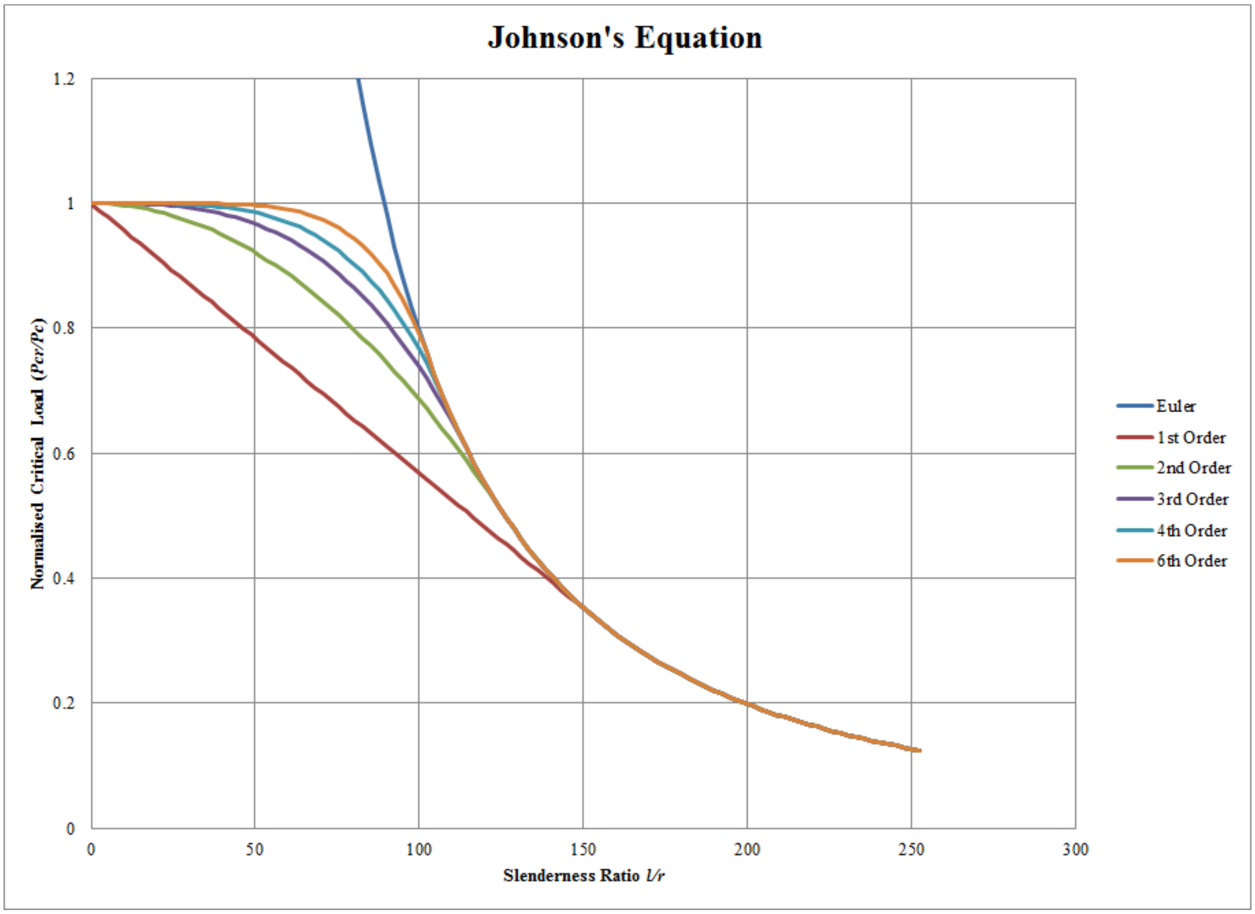

Figure 1. Johnson’s Equation

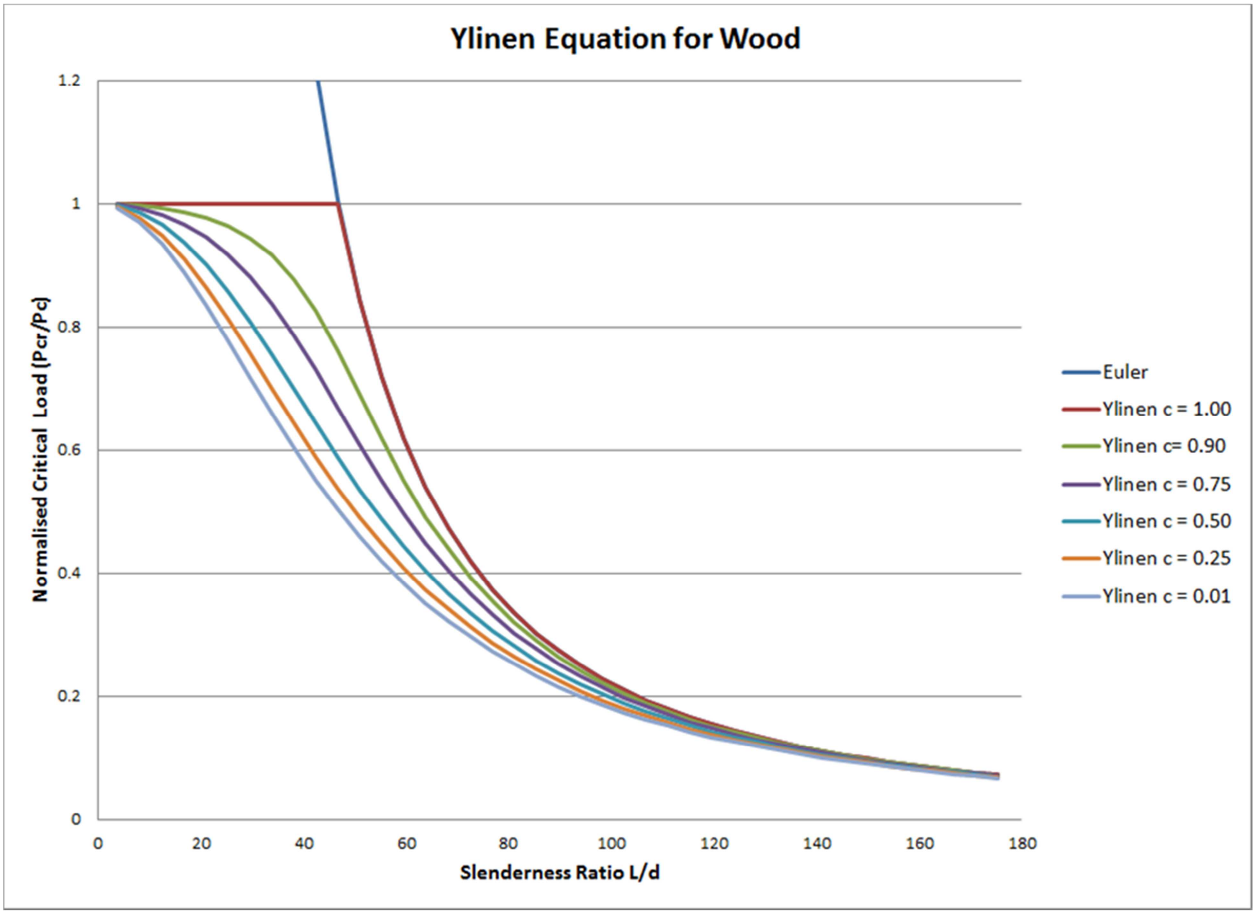

Figure 2. Ylinen’s Equation

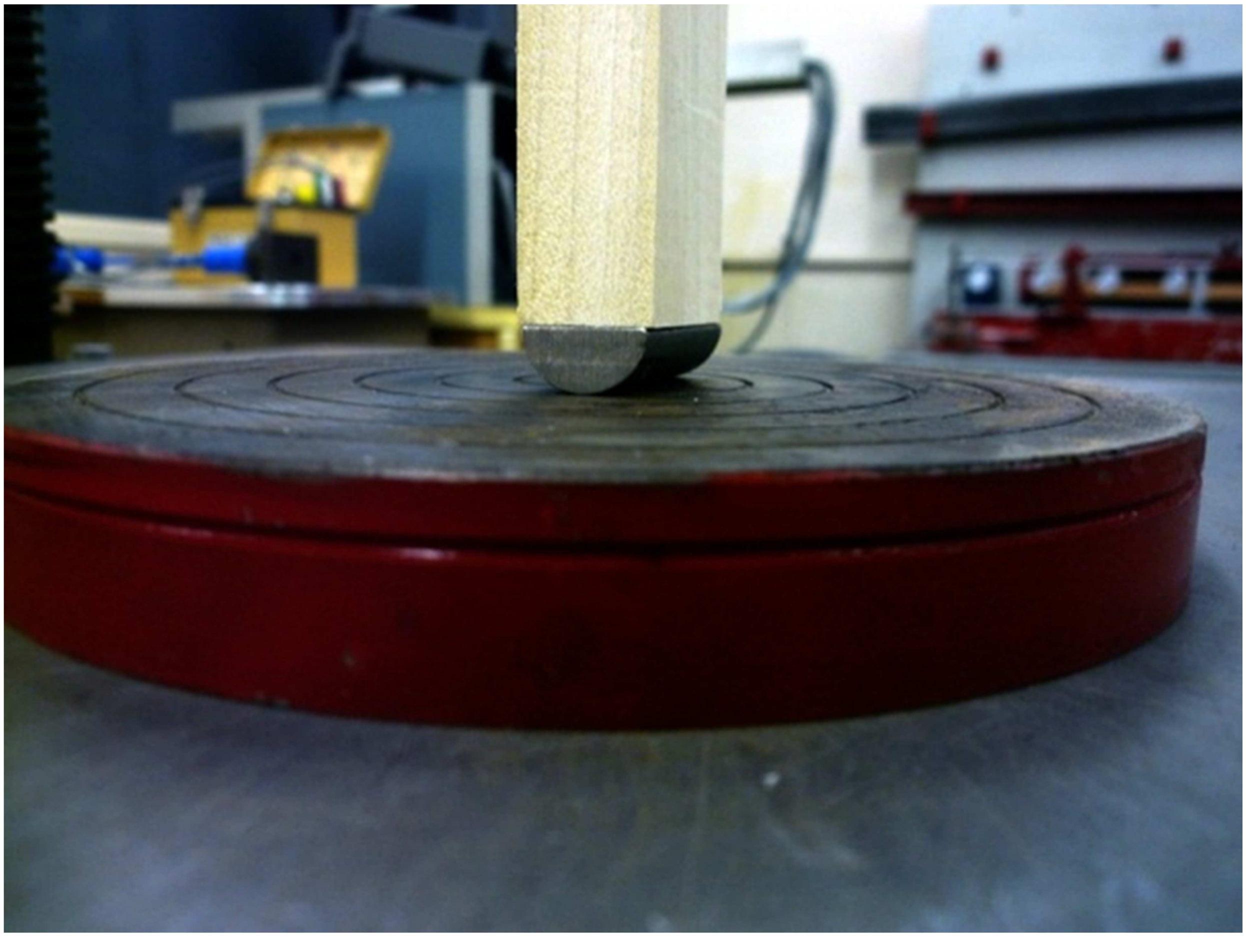

Figure 3. Column Test Pin Support

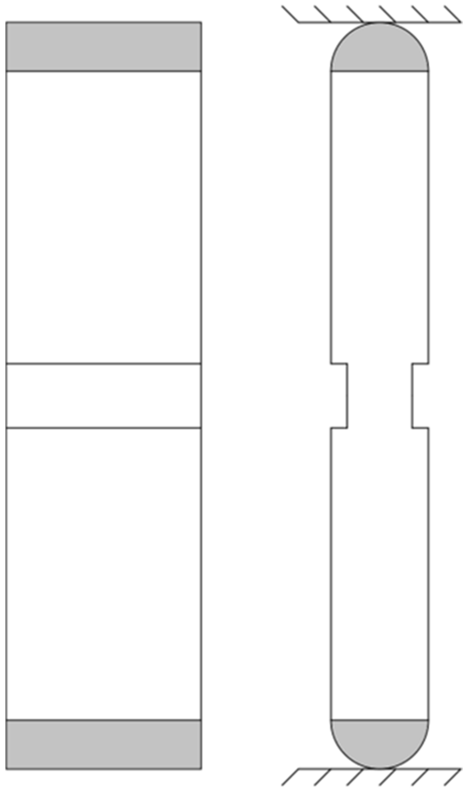

Figure 4. Typical Column Test Specimen with Notch

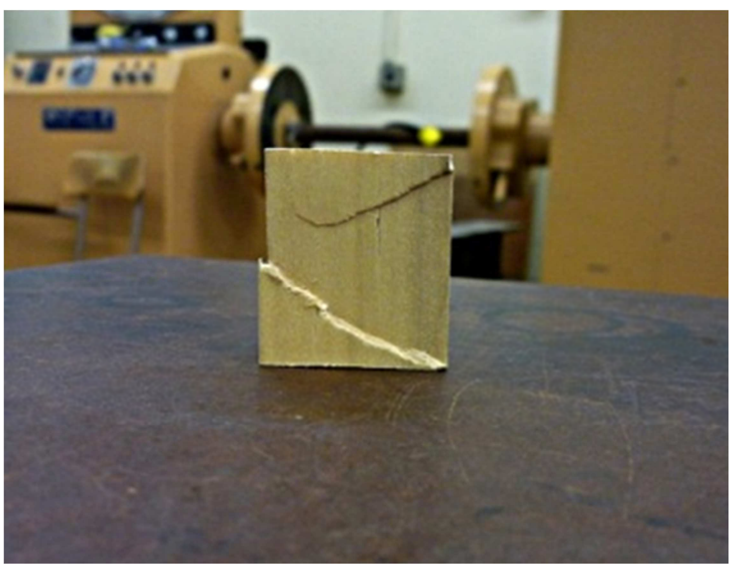

Figure 5. Crush Block Test No Notch

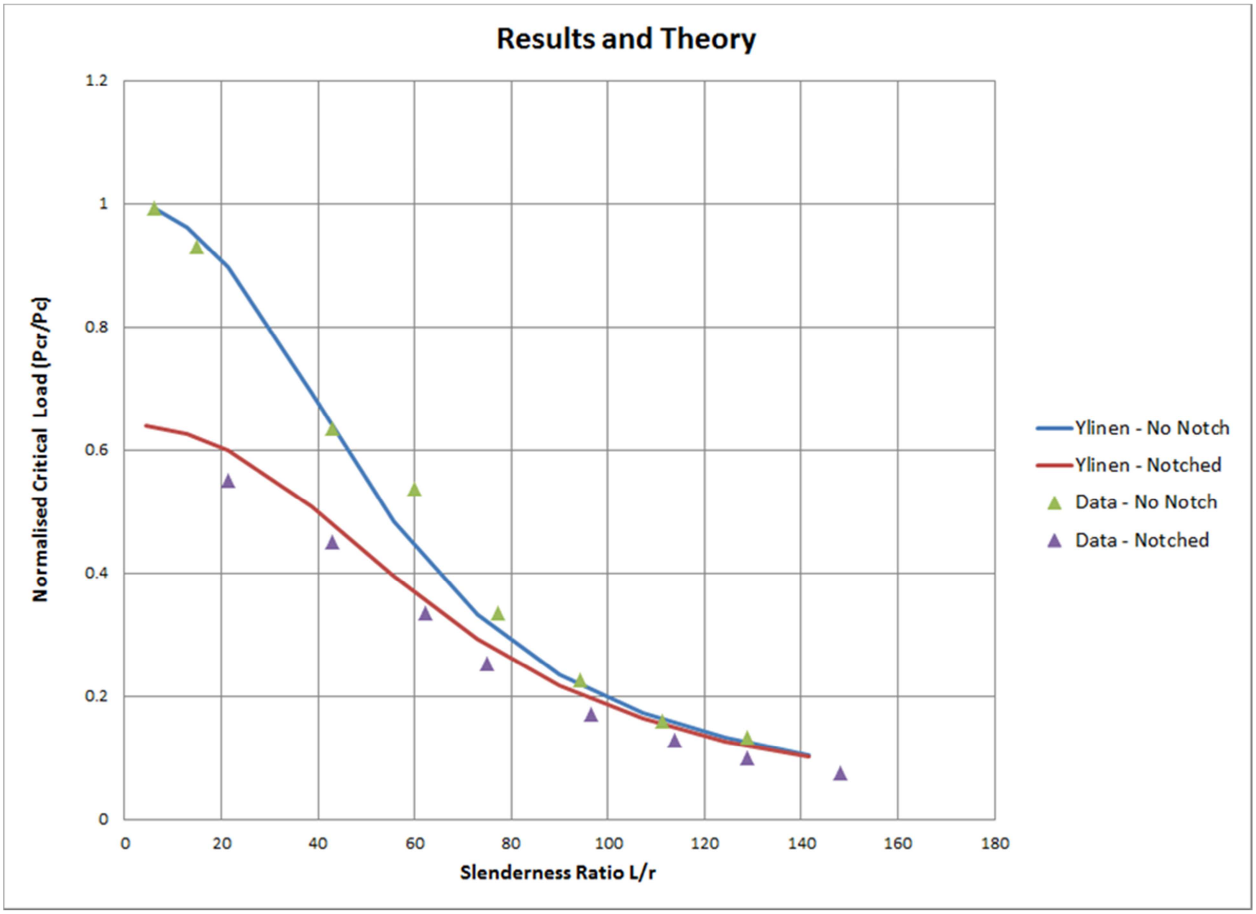

Figure 6. Results